Mathematical fluency in high school students

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This article presents the results of a study of mathematical fluency in high school students. We provide a definition of mathematical fluency and illustrate the relevance of the research by presenting an overview of studies examining mathematical fluency development and its relationship with success in mathematical disciplines.

A computerized test “Problem Verification Task” (Tosto et al., 2013) was administered to 692 high school students from one public secondary school (grades 9/10/11: n = 336/210/146) in the Moscow region. The stimuli consisted of 48 elementary arithmetic equations along with answer options. To indicate a correct answer, participants were instructed to press the corresponding key on the keyboard as quickly as possible.

Two-way ANOVA was used to estimate grade and sex similarities and differences in mathematical fluency at the high school level.

The current study has two primary findings: (1) students differed in math fluency across grades, and (2) there were no sex differences in mathematical fluency at the high school level. ANOVA exhibited significant differences in mathematical fluency among all three groups of students at grades 9, 10 and 11 with a 19% effect size. These results may be associated with the accumulating effects of the educational process: high school students in each subsequent year of schooling demonstrate a higher level of mathematical fluency on average compared to the previous year. At the same time, we observed no sex differences in mathematical fluency at the high school level. The results are discussed in terms of educational effects.

Keywords: mathematical fluency, mathematical achievement, education, high school age, sex differences
Introduction

The concept of fluency in psychology is traditionally associated with the dynamic characteristics of the thinking process and is used as a psychometric index in a range of diagnostic tests, for example, tests of creative thinking (Runco et al., 2010). In this context, fluency is defined as a specific combination of accuracy and response speed, and it characterizes competence in certain fields (Binder, 1996; Therrien, 2004). In experimental studies, the concept of fluency was studied primarily in relation to reading and is associated with the development of basic reading skills (Vygotsky, 1983; Kuhn et al., 2010; Hasbrouck, Tindal, 2006; Therrien, 2004; Fuchs et al., 2001 и др.). L.S. Vygotsky wrote that reading speed is directly related to depth of understanding and text comprehension. In particular, Vygotsky noted that “... during fast reading, comprehension turns out to be better ... and the comprehension speed is related to faster pace reading” (Vygotsky, 1983, p. 192).

However, the focus of attention has recently shifted toward mathematical fluency, which is defined as the ability to perform basic mathematical operations quickly and accurately; this ability leads to success in mathematical disciplines (Singer-Dudek, Greer, 2005; Floyd et al., 2003). Fluency in arithmetic is essential for the success of students in everyday life because it serves as a foundation for mathematical applications such as time and money (Smith et al., 2011). There are three main reasons to investigate mathematical fluency. First, fluency in basic mathematical operations is essential to mastering higher-order mathematical skills, such as multi-step problems in algebra (Smith et al., 2011). Second, due to the lack of effort required and more frequent successes, schoolchildren who respond automatically typically have less anxiety in math (Poncy et al., 2007). Third, high levels of mathematical fluency are maintained longer, and students are better able to resist distractions and stay on task longer (Rhymer et al., 2000).

Indeed, experimental studies aimed at optimizing the process of assimilation and further application of basic mathematical skills are of great social and practical value in terms of the requirements of national educational standards, daily activities (Codding et al., 2009), and development of the knowledge-intensive sectors of society (Butterworth, Kovas, 2013).

Studies show that not only people with low levels of cognitive development experience difficulties in learning mathematics (Geary et al., 2000; Siegel, 1988). One reason for lack of success in mathematics might be low mathematical fluency (Binder 1996; Ramos-Christian et al., 2008). This relationship might stem from the incorrect distribution and/or limitations of cognitive powers; in other words, a person spends most of his cognitive abilities at the primary stages of problem-solving, leaving only the minimum necessary to perform the more complicated steps that lead to a successful solution (Delazer et al., 2003).

It has been shown that while solving mathematical problems of increased complexity, for example, people with low mathematical fluency must first make maximum use of their cognitive resources (e.g., attention, memory) to perform basic arithmetic operations, limiting the use of those resources in further problem solving (Ramos-Christian et al., 2008; Skinner et al., 2005; Dehaene, 2011). In turn, individuals with high levels of mathematical fluency manage quickly and accurately to perform initial calculations, saving more cognitive resources for completing the
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It was also demonstrated that students with higher levels of mathematical fluency exhibit low levels of mathematical anxiety (Cates, Rhymer, 2003) and greater motivation for mathematical activities (Codding et al., 2009), contributing to higher achievement in the field.

The relationship between mathematical fluency and success in mathematical disciplines has led to increased interest in the development of educational technologies aimed at boosting mathematical fluency. There are many experimental studies focused on comparing the effectiveness of training programs aimed at improving mathematical fluency among students and school children (Hulac et al., 2012; Smith et al., 2011; Bramlett et al., 2010; Codding et al., 2009; Seethaler & Fuchs, 2005). Mathematical fluency is usually assessed with the mathematical fluency subtest of the Woodcock-Johnson achievement test (Woodcock et al., 2001).

Mathematical fluency is often associated with sex differences in mathematical achievement. It is reported that men perform mathematical calculations better and faster than women (Wai et al., 2009; Royer et al., 1999). However, a meta-analysis of sex differences in solving standardized arithmetic tasks, which was conducted with a sample of US students, did not show any differences between mean values for boys and girls (Hyde et al., 2008). However, this study reported somewhat greater variability indices for boys than for girls.

There are few studies of the etiology of mathematical fluency. One behavioral-genetic study was designed to evaluate the contributions of genetic and environmental factors to the relationship between mathematical fluency and other characteristics of mathematical success (Petrill et al., 2012; Malykh et al., 2012).

It should be noted that most existing studies examined developmental changes in mathematical fluency in elementary school children (e.g., Bartelet, 2014; Cowan et al., 2011; Geary, 2011; Carr & Alexeev, 2011). This finding may reflect a belief that mathematical fluency undergoes major development during elementary school years and does not improve significantly at an older age. However, high school math includes more complex problems, which provide practice in computation, so it is possible that mathematical fluency continues to improve at that age through experience.
The aim of this study is to examine age and sex differences in the mathematical fluency of Russian high school students. We propose that on average, high school students in each subsequent year of schooling will demonstrate significantly higher mathematical fluency compared to the previous year.

**Method**

**Sample**

Research participants included 692 schoolchildren in grades 9–11 of public secondary school aged from 14.17 to 18.67 years (Mean = 16.53; SD = .88). Students in grade 9 accounted for 48.6% of the total number of participants (336 students, 50% boys), grade 10 — 30.3% (210 students, 45.7% boys) and grade 11 — 22.5% (146 students, 39.7% boys). The students were recruited from one public secondary school in a greater-Moscow region. The education curriculum at the school follows a standardized state program.

**Measure**

All participants completed a computerized test «Problem Verification Task» (PVT test). The stimuli consist of 48 arithmetic problems with a proposed answer (Tosto et al., 2013). All problems included elementary arithmetic equations, for example ‘8 : 2 = 4’, ‘12 + 50 = 62’, ‘44 – 18 = 24’, ‘13 × 4 = 47’, etc. In half of the trials, the answer is correct; in the other half, the answer is wrong. Every item is presented in the same format: a mathematical equation is shown at the top of the screen, and the keys ‘Right’, ‘Wrong’ and ‘I don’t know’ are shown beneath a mathematical equation. The timer is shown in the left corner of the screen. The image below shows the example items with a correct (a) and an incorrect answer (b) from the PVT test (see Fig. 1).

![Example of «Problem Verification Task» items](image)

**Figure 1.** Example of «Problem Verification Task» items

Participants decide whether the answer proposed is correct as quickly as possible by pressing the corresponding keys on the keyboard. The task starts with instructions and a practice trial that includes two items. The “time-out” for a response in this task is 10 seconds. A time bar appears at the top of the screen to show the
participant the time remaining. After a response is given, the problem is presented with no delay. It is possible to interrupt the task halfway-through and resume it later. The task records the accuracy and reaction time of a correct response.

Data collection was conducted in the educational institution, strictly following the protocol and under the constant supervision of an experimenter. Analysis of the results was carried out using anonymized personal data.

Results
The paper analyzed the mathematical fluency score, calculated as the number of correct responses, with a limit of 10 seconds for each task. First, the data were analyzed in terms of norms and distributions for the PVT test at each grade. Table 1 presents descriptive statistics by grade.

Table 1. Descriptive statistics for Problem Verification Task

<table>
<thead>
<tr>
<th>Grade</th>
<th>Males</th>
<th>Females</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>34.81 (7.9)</td>
<td>34.24 (6.8)</td>
</tr>
<tr>
<td>10</td>
<td>40.80 (5.8)</td>
<td>39.49 (4.5)</td>
</tr>
<tr>
<td>11</td>
<td>42.28 (4.8)</td>
<td>41.89 (3.7)</td>
</tr>
</tbody>
</table>

Table 1 shows the average number of correct answers for the PVT test. The lowest possible score on the test is 0; the maximum possible is 48. As illustrated in Table 1, mean scores for mathematical fluency increased with years of schooling. A decrease in the standard deviation in grade 11 indicates a trend towards concentration and regularity of mathematical fluency scores in students at grade 11. The male students in each grade demonstrated, on average, higher means and larger standard deviations than female students.

Second, we investigated sex and age differences in mathematical fluency at high school age. Effects of a year of schooling (grades 9, 10, 11) and sex (male, female) were assessed by two-way ANOVA. The dependent variable was PVT test scores.

To test the hypothesis that all dependent variables have the same variance, we used Levene’s test for equality of variances. The $p$-value was below .05, indicating variance inequality on the test indicator.

Table 2 shows that the effect of the factor «Years of schooling» was statistically significant for mathematical fluency ($\eta^2 = .19, p = .00$). The results of multiple com-

Table 2. Inferential statistics for Problem Verification Task

<table>
<thead>
<tr>
<th>Effect</th>
<th>SS</th>
<th>F</th>
<th>p</th>
<th>$\eta^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>‘Years of Schooling’</td>
<td>6208.42</td>
<td>80.72</td>
<td>.00</td>
<td>.19</td>
</tr>
<tr>
<td>‘Sex’</td>
<td>84.43</td>
<td>2.20</td>
<td>.14</td>
<td>.00</td>
</tr>
<tr>
<td>Effects interaction</td>
<td>22.60</td>
<td>.29</td>
<td>.75</td>
<td>.00</td>
</tr>
</tbody>
</table>
parisons with the Bonferroni correction showed significant differences between all the analyzed groups \( (p < .01) \). At the same time, the lowest mean value was shown in the group of students in grade 9 (Mean = 34.53, SD = 7.4). The best results on the PVT test were demonstrated by students in grade 11 (Mean = 41.04, SD = 4.2). The means and standard deviations for male and female students are presented in Table 1.

The gender effect was not significant for mathematical fluency \( (p > .05) \). Thus, males and females do not differ in mathematical fluency at high school age.

**Discussion**

The present study investigated the age and sex differences in mathematical fluency at high school age.

On one hand, the results may show the cumulative effects of the educational process — on average, students in each subsequent academic year have had a greater number of classes and mathematical practice out of school compared to previous grade levels. It is especially important for students in grade 11, when all Russian students take the Unified State Exam, which includes math. Indeed, frequent use of elementary or complex mathematical operations can improve mathematical fluency (Hulac et al., 2012; Bramlett et al., 2010). For example, it was demonstrated that the method “Cover, Copy, and Compare”, which is based on behaviorism, is an effective tool in improving mathematical fluency (Poncy et al., 2010; Skinner et al., 1997). A statistically significant difference was found between groups with different levels of mathematical fluency in terms of the effectiveness of their spatial ability and levels of visuospatial memory (Tikhomirova et al., 2013). These results demonstrate that mathematical fluency can be learned through education (as in this study) or through specially designed training sessions (in accordance with the theoretical analysis presented above).

On the other hand, the improvement of mathematical fluency from grades 9 to 11 may be associated with maturational changes. However, numerous studies demonstrate that education is one of the most important factors in age-related cognitive development (e.g., Brinch & Galloway, 2012). In the current study, we used a cross-section associational design and have not analyzed age-related differences.

Regarding sex differences, males and females do not differ in mathematical fluency in high school. It should be noted that in one of our previous studies on a small sample of Russian high school students, we also found that the gender effect on mathematical fluency was not statistically significant (Tikhomirova & Kovas, 2013). The results of this study conducted on a larger sample showed no sex differences in mathematical fluency in Russian high school students. This result does not replicate the results of the study of sex differences in mathematical fluency in American students (Geary et al., 2000). It was reported that, compared to girls, boys perform significantly better on elementary arithmetic tasks. This discrepancy in results may be due to cross-cultural differences in education systems.

In general, existing studies have found a bi-directional relationship between mathematical fluency and mathematical achievement. A large body of studies
have reported that mathematical fluency is a predictor of mathematical success (e.g., Singer-Dudek, Greer, 2005; Smith et al., 2011). At the same time, studies describing the effects of training sessions have demonstrated effects of mathematical exercises on mathematical fluency (Hulac et al., 2012; Bramlett et al., 2010). In other words, mathematical fluency and success in mathematics may be reciprocally interrelated. Longitudinal research is needed to establish the precise developmental course of the links between mathematical fluency and mathematical achievement at school age.

At the same time, according to international mathematical ratings (e.g., PISA, TIMMS), there are stable, cross-cultural differences in mathematical achievement in schoolchildren. Therefore, it seems promising to conduct a cross-cultural, longitudinal study in different grades to detect individual and cross-cultural differences in the structure of the relationships between mathematical fluency and mathematical achievement and to determine the specific cultural, linguistic, and educational factors contributing to the observed cross-cultural differences.

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References


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